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Entropic Force between Two Distant Black Holes in a Background Temperature

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Abstract: **We use the Newton's law of gravitation as an entropic force to study some thermodynamical properties of a system of two Schwarzschild black holes which are immersed in a background temperature. For this we assume the two black holes are at large separation distance. We extract entropy, free energy and internal energy of the system. Positive definiteness of the free energy leads to a limit on the black holes temperatures.**

Keyword: **Entropic force; black hole; entropy; free energy; internal energy.**

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1. Introduction

The hypothesis of entropic force nature of gravity has a history that goes back to the investigation on black hole thermodynamics by Bekenstein and Hawking [1, 2]. These researches show a deep and fundamental relationship between thermodynamics and gravitation. In 1995 Jacobson demonstrated that the Einstein equation of general relativity can be derived by combining the equivalence principle with general thermodynamics considerations [3]. Subsequently, other physicists have further explored the link between gravity and entropy [4]. E. Verlinde initiated a conceptual theory that describes gravity as an entropic force [4]. This theory is a combination of the holographic principle with the thermodynamics approach to gravity. This implies that gravity is not a fundamental force, but an emergent phenomenon which arises from the statistical behavior of microscopic degrees of freedom encoded on a holographic screen. Immediately after the idea of the entropic force, some applications have been carried out. In this article we consider a system of two Schwarzschild black holes with the masses M1 and M2 which is embedded in a background space (i.e. an infinite heat bath) with the temperature T. According to the Verlinde's proposal this temperature is associated with some degrees of freedom which are fundamental in an underlying theory. The gravitational force then reflects the rearrangement of these degrees of freedom. However, combining the Newton's law of gravitation as entropic force with the entropic force law, we obtain the entropy of the system. For receiving this entropy we introduce the Bekenstein-Hawking entropy for each isolated black hole, i.e. when the separation of them is infinite. Using this entropy function we derive the entropy of a special model of the universe which reveals that for an expanding universe the entropy of the background space is increasing. The entropy function also enables us to obtain the Helmholtz free energy and internal energy of the system. Imposing positivity on the free energy leads to a condition on the temperatures of the black holes, and consequently a lower bound for the internal energy. Note that we do not take into account the quantum effects. In addition, we use the Newton's law of gravitation but not the general relativity. Therefore, our analysis is valid only for the distant black holes, that is, the separation between them should be very larger than the sum of their Schwarzschild radii.

2 Entropy of the System

We consider two classical black holes with zero charge and zero angular momentum as our setup. According to the Ref. [4] the entropic force is given by

$$\vec{F}(\vec{r}) = T\vec{\nabla} S, \tag{1}$$

where the vector $\vec{r}$ indicates distance between the centers of the black holes with the direction from the black hole M_1 toward the black hole M_2, T is the temperature of the background space (i.e. an infinite heat bath) that is supposed to be constant and S is the entropy of the black holes system. Note that we assume the motions of the black holes do not change the background temperature T. In addition, in our setup only the far away black holes can be considered. This helps us to ignore the deformation of the horizon area of each black hole, which occurs in the presence of the gravitational field of the other black hole and leads to the enhancement of the horizon area.

We apply the Newton's law of gravitation $\vec{F}_{\rm g} = -G\frac{M_1 M_2}{r^3}\vec{r}$ as the entropic force [4]. This implies that our calculations are valid for the range $r \gg R_S^{(1)} + R_S^{(2)}$, where $R_S^{(1)}$ and $R_S^{(2)}$ are Schwarzschild radii of the black holes. However, we receive the equation

$$T\mathrm{d}S = -G\frac{M_1 M_2}{r^2}\mathrm{d}r. \tag{2}$$

Integration of this equation gives the entropy as in the following

$$S(r) = S_0 + \frac{G}{T}\frac{M_1 M_2}{r}. \tag{3}$$

The constant S_0 is the entropy of the system when the black holes distance is infinite. In this case the system contains two isolated black holes, and hence S_0 is sum of the entropies of the black holes $S_0 = S_{\rm BH}^{(1)} + S_{\rm BH}^{(2)}$. The entropy of an isolated black hole is given by the Bekenstein-Hawking formula $S_{\rm BH} = \frac{k_{\rm B} A}{4L_{\rm P}^2}$, where $L_{\rm P} = \sqrt{G\hbar/c^3}$ is the Planck length and $k_{\rm B}$ is the Boltzmann constant. The surface at the Schwarzschild radius $R_{\rm S} = 2GM/c^2$ acts as event horizon with the area $A = 4\pi R_{\rm S}^2$. These reveal the entropy of the system as

$$S(r) = 4\pi k_{\rm B}\left(\frac{M_1^2 + M_2^2}{M_{\rm P}^2}\right) + \frac{G}{T}\frac{M_1 M_2}{r}, \tag{4}$$

where $M_{\rm P} = \sqrt{\hbar c/G}$ is the Planck mass.

We observe that the entropy at finite separation is larger than the isolated case. In other words, for decreasing distance this entropy increases. The second law of thermodynamics implies that when the entropy of the system changes the entropy of the rest part of it, i.e. the infinite thermal heat bath, also changes in a manner that total entropy of the whole heat bath (including the two black holes) to be enhanced.

2.1 Entropy of the universe

At first consider N spherical masses $\{M_i|i = 1, 2, \cdots, N\}$. These objects mutually interact. The entropy associated with the i^{th} and j^{th} of them is given by the Eq. (3),

$$S_{ij} = S_i + S_j + \frac{G}{T}\frac{M_i M_j}{r_{ij}}, \tag{5}$$

where S_i and S_j are the entropies of these objects when they are isolated. This equation specifies the total entropy of these N objects

$$S = \frac{1}{N-1}\sum_{i,j=1,j>i}^{N} S_{ij}\ , \qquad N \geq 2. \tag{6}$$

When all r_{ij}s are infinite all objects are isolated and hence we receive the entropy $S = \sum_{i=1}^{N} S_i$, as expected.

Now we assume that the universe is suffused of various spherical objects in a background space with the temperature T. Therefore, the Eqs. (5) and (6) define the entropy of this model of the universe as in the following

$$S_{\text{Universe}} = S_{\text{bs}} + \sum_{i=1}^{N} S_i + \frac{1}{N-1}\frac{G}{T}\sum_{j>i}^{N}\frac{M_i M_j}{r_{ij}}, \tag{7}$$

where N is the number of objects in the universe and S_{bs} represents entropy of the background space. If the i^{th} object is a black hole its entropy S_i is given by the Bekenstein-Hawking formula, and if it is a non-black hole (i.e. all known configurations of matter, including: gas clouds, galaxies, ordinary stars and neutron stars) the bound $S_i < \lambda A_i^{3/4}$ is on its entropy [5]. Standard estimations suggest that the entropy of our universe is dominated by black holes.

Since our universe is expanding, approximately all r_{ij}s are increasing which reduce the mutual entropy of the objects. According to the second law of thermodynamics this demonstrates that enhancement of the universe entropy is due to the increment of the background space entropy.

3 Free energy and internal energy of the system

The Helmholtz free energy $\mathcal{F}$ and the internal energy U of the system are related to the entropy through the equation

$$TS = U - \mathcal{F}. \tag{8}$$

Comparing this relation with the Eq. (4) specifies the free and internal energies as in the following

$$\mathcal{F} = 4\pi k_{\mathrm{B}} T \Big(\frac{M_1^2 + M_2^2}{M_{\mathrm{P}}^2} \Big) - G \frac{M_1 M_2}{r}, \tag{9}$$

$$U = 8\pi k_{\mathrm{B}} T \Big(\frac{M_1^2 + M_2^2}{M_{\mathrm{P}}^2} \Big). \tag{10}$$

For decomposing the quantity TS into these parts we imposed the definition of the free energy as follows. The Helmholtz free energy is a thermodynamical potential which measures the useful work obtainable from the system at constant temperature. Therefore, the negative of the difference in $\mathcal{F}$ is equal to the maximum amount of extractable work W. The Eq. (9) satisfies this condition, i.e., $-\mathrm{d}\mathcal{F} = \mathrm{d}W$, where $\mathrm{d}W = \overrightarrow{F}_{\mathrm{g}} \cdot \mathrm{d}\overrightarrow{r}$ is the work which is done by the gravitational force. In addition, since for any spontaneous process the Helmholtz free energy must decrease, the Eq. (9) is an expected result.

Furthermore, the maximum value of the free energy is storaged at infinite distance

$$\mathcal{F}_{\max} = 4\pi k_{\mathrm{B}} T \Big(\frac{M_1^2 + M_2^2}{M_{\mathrm{P}}^2} \Big). \tag{11}$$

Let the black holes move toward each other and the system begins its working. Thus, the initial free energy $\mathcal{F}_{\max}$ begins to decrease and changes to work. At the distance r the free energy $\mathcal{F}_{\max}$ reduces to (9), and hence, amount of the extracted work is $\mathcal{F}_{\max} - \mathcal{F}(r) = GM_1M_2/r$. Decreasing of the free energy is consistent with the second law of thermodynamics, which implies that there is a tendency to the dissipation (energy loss) of the mechanical energy (motion), hence, the mechanical movement of the black holes will run down as work, which is converted to heat [6].

The internal energy can be written in the form

$$U = \Big(\frac{T}{T_H^{(1)}} \Big) M_1 c^2 + \Big(\frac{T}{T_H^{(2)}} \Big) M_2 c^2 \,, \tag{12}$$

where $T_{\rm H} = \frac{\hbar c^3}{8\pi G k_{\rm B} M}$ is the Hawking radiation temperature of a black hole with the mass M. The first terms of the entropy and free energy also can be written in the feature of this equation. However, the Eq. (12) demonstrates that contribution of each black hole to the internal energy is proportional to its rest energy, and the proportionality coefficients depend on the background temperature and temperatures of the black holes. Since for the zero temperature the internal energy vanishes we conclude that the rest energies are not a portion of thermodynamical internal energy. One can define a total internal energy, that is sum of the thermodynamical internal energy and the rest energies.

3.1 An upper bound on the black hole temperature

For all values of the masses, background temperature and distance the entropy and internal energy are positive, while the free energy may be negative. We impose the positivity condition on the free energy. If we consider the condition $\mathcal{F}|_{r=R_S^{(1)}+R_S^{(2)}} > 0$ we shall sure that for any value of the distance r, in the allowed range $r \gg R_S^{(1)} + R_S^{(2)}$, the free energy will be positive. This leads to the following limitation

$$\bar{T}_H < 2\left(\frac{M_1^2 + M_2^2}{M_1 M_2}\right) T, \qquad (13)$$

where the Hawking temperature $\bar{T}_H$ is corresponding to a black hole with the average mass $(M_1 + M_2)/2$. Consequently this inequality gives a lower bound for the internal energy

$$U > \mu c^2, \qquad (14)$$

where $\mu = M_1 M_2/(M_1 + M_2)$ is the reduced mass of the black holes. For black holes with equal masses we receive

$$T_H < 4T. \qquad (15)$$

In this case each black hole can possess at most the temperature $4T$.

4 Conclusions

The calculations in this article yield a novel picture of two black holes system, immersed in a background heat bath, which interact via the gravitational force from the perspective of the Verlinde's conjecture, that describes gravity as an entropic force. We obtained the entropy, free energy and internal energy of the system. This system helps us to calculate the entropy of a special model for the universe. In this model for an expanding universe the mutual entropy of the astronomical objects is decreasing and the entropy of the background space is increasing. We observed that contribution of each black hole to the thermodynamical internal energy is specified by its rest energy, the corresponding Hawking temperature and background temperature. The internal energy also enabled us to redefine the mass of a black hole which lives in a background temperature.

Positive definiteness of the free energy imposed a restriction on the black holes temperatures, which defines a lower bound for the internal energy. The temperature of identical black holes cannot exceed the value $4T$.

References

[1] J.D. Bekenstein, "*Black holes and entropy*", Phys. Rev. D7 (1973) 2333.

[2] S.W. Hawking , "*Particle creation by black holes*", Comm. Math. Phys. 43 (1975) 199.

[3] T. Jacobson, "*Thermodynamics of spacetime: The Einstein equation of state*", Phys. Rev. Lett. 75 (1995) 1260, arXiv: gr-qc/9504004.

[4] E.P. Verlinde, "*On the Origin of Gravity and the Laws of Newton*", JHEP 1104:029, 2011, arXiv: 1001.0785 [hep-th].

[5] G. 't Hooft, "*Dimensional reduction in quantum gravity*", arXiv: gr-qc/9310026.

[6] William Thomson "*On a Universal Tendency in Nature to the Dissipation of Mechanical Energy*", Proceedings of the Royal Society of Edinburgh for April 19, 1852; also Philosophical Magazine, Oct. 1852.